Character table for S_3

$\underline{G = S_3}$: conjugacy classes

$\{\underline{e}\}$ $\{\underline{(1\,2)}, (2\,3), (1\,3)\}$ $\{\underline{(1\,2\,3)}, (1\,3\,2)\}$

- $S_3/[S_3, S_3] \simeq S_3/\langle (1\,2\,3) \rangle$

	e [1]	$(1\,2)$ [3]	$(1\,2\,3)$ [2]
χ_1	1	1	1
χ_2	1	-1	1

$$\langle \chi_1, \chi_2 \rangle = \frac{1}{6}\left[(1)(1) + 3(1)(-1) + 2(1)(1)\right] = 0$$

$\rho : S_3 \to GL_3(\mathbb{C})$ std rep

$$\rho_e = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \rho_{(12)} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \rho_{(123)} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

	$\overset{1}{e}$	$\overset{3}{(12)}$	$\overset{2}{(123)}$
χ_1	1	1	1
χ_2	1	-1	1
χ_ρ	3	1	0

$$\langle \chi_\rho, \chi_\rho \rangle = \frac{1}{6}\left[(3)(\overline{3}) + 3(1)(\overline{1}) + 2(0)(\overline{0})\right] = 2$$

$$\langle \chi_1, \chi_\rho \rangle = \frac{1}{6}\left[(1)(\overline{3}) + 3(1)(\overline{1}) + 2(1)(\overline{0})\right] = 1$$

$$\langle \chi_2, \chi_\rho \rangle = \frac{1}{6}\left[(1)(\overline{3}) + 3(-1)(\overline{1}) + 2(1)(0)\right] = 0$$

so $\rho \sim \chi_1 \oplus \psi$

<u>Know</u>: $\underline{\chi_\rho} = \underline{\chi_1} \oplus \chi_\psi$

	1	3	2
	e	(12)	(123)
χ_1	1	1	1
χ_2	1	-1	1
$\chi_3 = \chi_\psi$	2	0	-1

$\langle \chi_\psi, \chi_\psi \rangle = 1$

Also have $\chi_2 \cdot \psi \Rightarrow \chi_{\chi_2 \psi} = \chi_2 \cdot \chi_3$